Paper Reference(s)

## 6664/01

## Edexcel GCE

## Core Mathematics C2

## Gold Level G3

## Time: 1 hour 30 minutes

$\frac{\text { Materials required for examination }}{\text { Mathematical Formulae (Green) }} \quad \frac{\text { Items included with question papers }}{\mathrm{Nil}}$

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

Suggested grade boundaries for this paper:

| A* | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 67 | 58 | 48 | 39 | 30 | 23 |

1. 

$$
\mathrm{f}(x)=2 x^{3}-3 x^{2}-39 x+20
$$

(a) Use the factor theorem to show that $(x+4)$ is a factor of $\mathrm{f}(x)$.
(b) Factorise $\mathrm{f}(\mathrm{x})$ completely.
(4)

June 2008
2. A circle $C$ with centre at the point $(2,-1)$ passes through the point $A$ at $(4,-5)$.
(a) Find an equation for the circle $C$.
(b) Find an equation of the tangent to the circle $C$ at the point $A$, giving your answer in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.

May 2015
3. (a) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of $x$, giving each term in its simplest form.
(b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

January 2008
4.


## Figure 1

An emblem, as shown in Figure 1, consists of a triangle $A B C$ joined to a sector $C B D$ of a circle with radius 4 cm and centre $B$. The points $A, B$ and $D$ lie on a straight line with $A B=5 \mathrm{~cm}$ and $B D=4 \mathrm{~cm}$. Angle $B A C=0.6$ radians and $A C$ is the longest side of the triangle $A B C$.
(a) Show that angle $A B C=1.76$ radians, correct to three significant figures.
(b) Find the area of the emblem.

January 2010
5. (a) Find the positive value of $x$ such that

$$
\begin{equation*}
\log _{x} 64=2 \tag{2}
\end{equation*}
$$

(b) Solve for $x$

$$
\log _{2}(11-6 x)=2 \log _{2}(x-1)+3
$$

6. 



Figure 2
The points $P(-3,2), Q(9,10)$ and $R(a, 4)$ lie on the circle $C$, as shown in Figure 2.
Given that $P R$ is a diameter of $C$,
(a) show that $a=13$,
(b) find an equation for $C$.
7. A geometric series has first term 5 and common ratio $\frac{4}{5}$.

Calculate
(a) the 20th term of the series, to 3 decimal places,
(b) the sum to infinity of the series.

Given that the sum to $k$ terms of the series is greater than 24.95,
(c) show that $k>\frac{\log 0.002}{\log 0.8}$,
(d) find the smallest possible value of $k$.
8. (a) Solve for $0 \leq x<360^{\circ}$, giving your answers in degrees to 1 decimal place,

$$
\begin{equation*}
3 \sin \left(x+45^{\circ}\right)=2 . \tag{4}
\end{equation*}
$$

(b) Find, for $0 \leq x<2 \pi$, all the solutions of

$$
2 \sin ^{2} x+2=7 \cos x
$$

giving your answers in radians.
You must show clearly how you obtained your answers.
9.


Figure 3
Figure 3 shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height $h \mathrm{~cm}$. The cross section is a sector of a circle. The sector has radius $r \mathrm{~cm}$ and angle 1 radian.

The volume of the box is $300 \mathrm{~cm}^{3}$.
(a) Show that the surface area of the box, $S \mathrm{~cm}^{2}$, is given by

$$
S=r^{2}+\frac{1800}{r} .
$$

(b) Use calculus to find the value of $r$ for which $S$ is stationary.
(c) Prove that this value of $r$ gives a minimum value of $S$.
(d) Find, to the nearest $\mathrm{cm}^{2}$, this minimum value of $S$.

## END

| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| $1 \text { (a) }$ <br> (b) | Attempt to find $\mathrm{f}(-4)$ or $\mathrm{f}(4) . \quad\left(\mathrm{f}(-4)=2(-4)^{3}-3(-4)^{2}-39(-4)+20\right)$ $(=-128-48+156+20)=0, \quad$ so $(x+4)$ is a factor. $\begin{aligned} & 2 x^{3}-3 x^{2}-39 x+20=(x+4)\left(2 x^{2}-11 x+5\right) \\ & \ldots \ldots(2 x-1)(x-5) \text { or equivalent } \end{aligned}$ | M1 <br> A1 <br> (2) <br> M1 A1 <br> M1 A1cso <br> (4) <br> [6] |
| 2 (a) (b) | $(x \mp 2)^{2}+(y \pm 1)^{2}=k, k>0$ <br> Attempts to use $r^{2}=(4-2)^{2}+(-5+1)^{2}$ <br> Obtains $(x-2)^{2}+(y+1)^{2}=20$ <br> Gradient of radius from centre to $(4,-5)=-2$ (must be correct) $\text { Tangent gradient }=-\frac{1}{\text { their numerical gradient of radius }}$ <br> Equation of tangent is $(y+5)=\frac{1}{2}^{\prime}(x-4)$ <br> So equation is $x-2 y-14=0$ | M1 <br> M1 <br> A1 <br> (3) <br> B1 <br> M1 <br> M1 <br> A1 <br> (4) |
| 3 (a) | $\begin{aligned} & \left(1+\frac{1}{2} x\right)^{10}=1+\binom{10}{1}\left(\frac{1}{2} x\right)+\binom{10}{2}\left(\frac{1}{2} x\right)^{2}+\binom{10}{3}\left(\frac{1}{2} x\right)^{3} \\ & \left.=1+5 x ;+\frac{45}{4} \text { (or } 11.25\right) x^{2}+15 x^{3}(\text { coeffs need to be these, i.e, simplified) } \end{aligned}$ <br> [Allow A1AO, if totally correct with unsimplified, single fraction coefficients) $\begin{aligned} & \left(1+\frac{1}{2} \times 0.01\right)^{10}=1+5(0.01)+\left(\frac{45}{4} \text { or } 11.25\right)(0.01)^{2}+15(0.01)^{3} \\ & =1+0.05+0.001125+0.000015 \\ & =1.05114 \quad \text { cao } \end{aligned}$ | M1 A1 <br> A1; A1 <br> (4) <br> M1 A1 <br> A1 <br> (3) <br> [7] |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 4 (a) | Either $\frac{\sin (A \hat{C} B)}{5}=\frac{\sin 0.6}{4}$ or $4^{2}=b^{2}+5^{2}-2 \times b \times 5 \cos 0.6$ <br> $\therefore A \hat{C} B=\arcsin (0.7058 \ldots)$ $\therefore b=\frac{10 \cos 0.6 \pm \sqrt{\left(100 \cos ^{2} 0.6-36\right)}}{2}$ <br> $=[0.7835 .$. or 2.358$]$ $=[6.96$ or 1.29$]$ <br> Use angles of triangle Use sine $/$ cosine rule with value <br> $A \hat{B} C=\pi-0.6-A \hat{C} B$ for $b \sin B=\frac{\sin 0.6}{4} \times b$ or <br> (But as $A C$ is the longest side so)  <br> $A \hat{B} C=1.76\left(^{*}\right)(3 \mathrm{sf})$ $\cos B=\frac{25+16-b^{2}}{40}$ <br> [Allow $\left.100.7^{\circ} \rightarrow 1.76\right]$ $(B u t$ as $A C$ is the longest side so $)$ <br> In degrees $A \hat{B} C=1.76(*)(3 \mathrm{sf})$ <br> $0.6=34.377^{\circ}, A \hat{C} B=44.9^{\circ}$  | M1 <br> M1 <br> M1, <br> A1 |
| (b) | $\lfloor C \hat{B} D=\pi-1.76=1.38\rfloor \quad$ Sector area $=\frac{1}{2} \times 4^{2} \times(\pi-1.76)=[11.0 \sim 11.1]$ $\frac{1}{2} \times 4^{2} \times 79.3$ is M0 <br> Area of $\triangle A B C=\frac{1}{2} \times 5 \times 4 \times \sin (1.76)=[9.8]$ or $\frac{1}{2} \times 5 \times 4 \times \sin 101$ <br> Required area $=$ awrt 20.8 or 20.9 or 21.0 <br> or gives $21(2 \mathrm{sf})$ after correct work. | M1 <br> M1 <br> A1 |
|  |  | (3) [7] |



| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 6 (a) | PQ: $\quad m_{1}=\frac{10-2}{9-(-3)}\left(=\frac{2}{3}\right)$ and $Q R: m_{2}=\frac{10-4}{9-a}$ | M1 |
|  | $m_{1} m_{2}=-1: \quad \frac{8}{12} \times \frac{6}{9-a}=-1 \quad a=13$ | M1 A1 |
|  | Alternative method (Pythagoras) Finds all three of the following $(9-(-3))^{2}+(10-2)^{2},\left(\text { i.e.208) }, \quad(9-a)^{2}+(10-4)^{2}, \quad(a-(-3))^{2}+(4-2)^{2}\right.$ | M1 |
|  | Using Pythagoras (correct way around) e.g. $a^{2}+6 a+9=240+a^{2}-18 a+81$ to form equation | M1 |
|  | Solve (or verify) for $a, a=13(*)$ | A1 |
|  |  | (3) |
| (b) | Centre is at ( 5,3$)$ | B1 |
|  | $\left(r^{2}=\right)(10-3)^{2}+(9-5)^{2}$ or equiv., or $\left(d^{2}=\right)(13-(-3))^{2}+(4-2)^{2}$ | M1 A1 |
|  | $(x-5)^{2}+(y-3)^{2}=65 \quad$ or $x^{2}+y^{2}-10 x-6 y-31=0$ | M1 A1 |
|  |  | (5) |
|  | Alternative method |  |
|  | Uses $(x-a)^{2}+(y-b)^{2}=r^{2}$ or $x^{2}+y^{2}+2 g x+2 f y+c=0$ | M1 |
|  | Eliminates second unknown | M1 |
|  | Obtains $g=-5, f=-3, c=-31$ or $a=5, b=3, \quad r^{2}=65$ | A1 A1 |
|  |  | B1 cao |
|  |  | (5) |
|  |  | [8] |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 7 (a) | $T_{20}=5 \times\left(\frac{4}{5}\right)^{19}=0.072$ | M1 A1 |
|  |  | (2) |
| (b) | $S_{\infty}=\frac{5}{1-0.8}=25$ | M1 A1 |
|  |  | (2) |
| (c) | $\frac{5\left(1-0.8^{k}\right)}{1-0.8}>24.95$ | M1 |
|  | $1-0.8^{k}>0.998$ or equivalent | A1 |
|  | $k \log 0.8<\log 0.002$ or $k>\log _{0.8} 0.002$ | M1 |
|  | $k>\frac{\log 0.002}{\log 0.8}$ | A1 cso |
|  |  | (4) |
| (d) | $k=28$ | B1 |
|  |  | (1) |
|  |  | [9] |



| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 9 (a) | (Arc length $=$ ) $r \theta=r \times 1=r$. Can be awarded by implication from later work, e.g. $3 r h$ or $(2 r h+r h)$ in the $S$ formula. (Requires use of $\theta=1$ ) (Sector area $=) \frac{1}{2} r^{2} \theta=\frac{1}{2} r^{2} \times 1=\frac{r^{2}}{2}$. Can be awarded by implication from later work, e.g. the correct volume formula. <br> (Requires use of $\theta=1$ ). | B1 B1 |
|  | Surface area $=2$ sectors +2 rectangles + curved face $\left(=r^{2}+3 r h\right) \quad$ (See notes below for what is allowed here) Volume $=300=\frac{1}{2} r^{2} h$ <br> Sub for $h: S=r^{2}+3 \times \frac{600}{r}=r^{2}+\frac{1800}{r}$ | M1 <br> B1 <br> A1 cso |
|  |  | (5) |
| (b) | $\frac{\mathrm{d} S}{\mathrm{~d} r}=2 r-\frac{1800}{r^{2}} \quad$ or $\quad 2 r-1800 r^{-2} \quad$ or $2 r+-1800 r^{-2}$ <br> $\frac{\mathrm{d} S}{\mathrm{~d} r}=0 \Rightarrow r^{3}=\ldots, \quad r=\sqrt[3]{900}$, or AWRT $9.7 \quad($ NOT -9.7 or $\pm 9.7)$ | M1 A1 <br> M1 A1 |
| (c) | $\frac{\mathrm{d}^{2} S}{\mathrm{~d} r^{2}}=\ldots$. and consider sign, $\frac{\mathrm{d}^{2} S}{\mathrm{~d} r^{2}}=2+\frac{3600}{r^{3}}>0$ so point is a minimum | (4) <br> M1 A1ft |
| (d) | $S_{\min }=(9.65 \ldots)^{2}+\frac{1800}{9.65 \ldots}$ <br> (Using their value of $r$, however found, in the given $S$ formula) <br> = 279.65... <br> (AWRT: 280) <br> (Dependent on full marks in part (b)) | M1 <br> A1 |
|  |  | (2) |
|  |  |  |

## Examiner reports

## Question 1

Part (a) of this question required the use of the factor theorem (rather than long division) and most candidates were able to show $f(-4)=0$. As in previous papers, a simple conclusion was expected. Many candidates failed to provide this.

The most popular strategy in part (b) was to use long division, dividing the cubic expression by $(x+4)$ to find the quadratic factor. Some candidates stopped at that stage and so could only gain a maximum of two marks, but of those who reached $2 x^{2}-11 x+5$ and went on to factorise this, the vast majority gained full marks. Less formal approaches to the division, including 'division by inspection', were occasionally seen and usually effective.

Candidates who solved $2 x^{2}-11 x+5=0$ gained neither of the final two marks until they produced the relevant factors, and then one of the factors was often left as $\left(x-\frac{1}{2}\right)$, which lost the final mark unless the factor 2 was included.

Some candidates went on to give 'solutions' $x=-4, x=5, x=\frac{1}{2}$, suggesting confusion over the meaning of 'factorise'.

## Question 2

Generally this question was well attempted and most candidates showed a sound understanding of the underlying concepts.

In part (a) most candidates recognised the correct structure for the equation of a circle and stated $(x-2)^{2}+(y+1)^{2}=r^{2}$ with only some slipping a power or the sign in between the brackets. A few used $(4,-5)$ as the coordinates for the centre and some gave a value for $r$ rather than for $r^{2}$ in the circle equation. Common sign errors in the calculation of the radius were seen, resulting in $r^{2}=40$. Some thought $A C$ gave them the diameter so continued to halve the distance found, losing the second method mark. A few candidates found the linear equation of the radius instead of the circle equation.

In part (b), the majority of candidates were able to find the gradient of the radius and proceeded correctly to find the gradient of the tangent and so went on to the final answer. A considerable number made sign errors in the calculation, or misapplied the formula for the gradient. In most cases the negative reciprocal was obtained and a correct method applied from this point, with most using the form $y-y_{1}=m\left(x-x_{1}\right)$ as the equation of a straight line rather than $y=m x+c$. Some failed to recognise the relationship between the gradient of the tangent and radius and continued to use their original gradient for the equation of the tangent. Some used the centre of the circle in the equation of the tangent, showing lack of understanding. Attempts at differentiation were seen in order to find the gradient, many resulting in few marks, due to the differentiation needed being beyond the scope of this module. A few tried to rearrange the equation to make $y$ the subject before differentiating but these attempts were rare. There were completely correct attempts at implicit differentiation seen but these were also rare. Some arithmetic errors were seen, especially if $y=m x+c$ was used. Many lost the final mark by not giving the equation of the line in the required form.

## Question 3

It was pleasing to see that most candidates could make some headway in part (a) and many candidates gained full marks. The usual errors of omitting brackets around $\frac{x}{2}$, and using $\left(\frac{10}{r}\right)$ for $\binom{10}{T}$, were seen, but not as frequently as on previous occasions. It was also common to see the coefficients of powers of $x$ not reduced to their simplest form. Solutions to part (b) were variable, with many candidates not able to find the appropriate value of $x$ to use; frequently 0.005 was substituted into a correct, or near correct expansion expression found in (a). Just writing the answer down, with no working at all, gained no marks.

## Question 4

(a) This was a discriminating question, as the method required two stages of solution. Candidates could either find the angle ACB using a correct form of the sine rule, then use angles of a triangle, or they could first find the length AC, then use the sine rule. Finding length AC was complicated (requiring a correct cosine rule and use of a quadratic formula) and the former method was easier. Weaker candidates tried to use Pythagoras, despite the triangle not being right angled, or used the sine rule wrongly and manipulated their answer to give the printed solution. Others assumed the printed answer and attempted verification, but this sometimes resulted in circular arguments and frequently the verification was not conclusive due to the angle being given correct to 3 sf. This verification method could earn a maximum of 2 out of 4 marks. Some candidates converted in and out of degrees, often successfully.
(b) Good candidates found the area of the triangle ABC and the area of the sector BCD and added these to give a correct answer. Weak candidates assumed that the emblem was a sector of radius 9 cm and angle 0.6 radians. Some made errors in their use of formulae and included pi erroneously, or neglected the $1 / 2$ factor. A few used the wrong angle in their formulae or indeed used the wrong formula, confusing arc length or area of a segment with area of a sector.

## Question 5

(a) Generally, both marks were scored easily with most candidates writing $x^{2}=64$ and $x=8$. Some included the -8 value as well, indicating that they were not always reading the finer details of the questions. However, quite a few attempts proceeded to $2^{x}=64$ leading to the most common incorrect answer seen of $x=6$. A small group squared 64 . Very few students attempted to change base in this part of the question.
(b) Most candidates scored the first M mark by expressing $2 \log _{2}(x-1)$ as $\log _{2}(x-1)^{2}$ but many then failed to gain any further marks. It was not uncommon for scripts to proceed from $\log _{2}(11-6 x)=\log _{2}(x-1)^{2}+3$ to $(11-6 x)=(x-1)^{2}+3$, resulting in the loss of all further available marks.

A significant number of candidates seem to be completely confused over the basic log rules. Working such as $\log _{2}(11-6 x)=\log _{2} 11 / \log _{2} 6 x$ following $\log _{2}(11-6 x)=\log _{2} 11-\log _{2} 6 x$ was seen on many scripts. Most candidates who were able to achieve the correct quadratic equation were able to solve it successfully, generally by factorisation, although some chose to apply the quadratic formula. There were a good number of completely correct solutions but the $x=-1 / 4$ was invariably left in, with very few candidates appreciating the need to reject it. Fortunately they were not penalised this time.

## Question 6

Part (a) caused much more of a problem than part (b). A large number of solutions did not really provide an adequate proof in the first part of this question. The original expected method, involving gradients, was the least frequently used of the three successful methods. Finding the three lengths and using Pythagoras was quite common although successful in a limited number of cases - there were many instances of equations being set up but abandoned when the expansion of brackets started to cause problems. Finding the gradient of QR as $-3 / 2$ and substituting to find the equation of the line for QR before using $\mathrm{y}=4$ to get a , was usually well done. Some used verification but in many cases this led to a circular argument.

In part (b) the centre was often calculated as $(8,3)$ or $(8,1)$ indicating errors with negative signs. There were several instances of $(5,4)$ arising from $(4+2) / 2$ being thought to be $4-$ maybe cancelling the 2 's? The length of PQ was usually correct but frequently thought to be the radius rather than the diameter. The equation of a circle was well known but weaker candidates in some cases took points on the circumference as the centre of the circle in their equation, showing lack of understanding.

## Question 7

Parts (a) and (b) of this question were very well done and the majority of candidates gained full marks here. A few, however, found the sum of 20 terms in part (a) rather than the $20^{\text {th }}$ term.

Only the very best candidates achieved full marks in part (c). The main difficulty was in dealing correctly with the inequality throughout the working. Often there were mistakes in manipulation and the division by $\log 0.8$ (a negative value) rarely resulted in the required 'reversal' of the inequality sign. Another common mistake was to say that $5 \times 0.8^{k}=4^{k}$. Despite these problems, many candidates were still able to score two or three marks out of the available four.

A surprising number of candidates made no attempt at part (d) and clearly did not realise they simply had to evaluate the expression in (c). Many failed to appreciate that $k$ had to be an integer.

## Question 8

Candidates were generally more successful in answering part (b) than part (a), but it was felt that a significant number of candidates were unsure about solving trigonometric equations and would have benefitted from a more methodical approach of either using a CAST diagram technique or solution curve technique. Although part (a) required answers in degrees and part (b) required answers in radians; this did not appear to be a problem for the majority of candidates.

In part (a), a significant number of candidates did not know how to deal with the 45 in $\sin \left(x+45^{\circ}\right)$ or with the order of operations to use. A fair number thought that $3 \sin \left(x+45^{\circ}\right)$ simplified to $3 \sin x+3 \sin 45^{\circ}$. Of those who correctly found $\sin ^{-1}\left(\frac{2}{3}\right)$, many were then unsure about how to proceed, with some candidates believing that $41.8^{\circ}$ was one of the values of $x$, whilst others subtracted 45 from this answer to achieve $-3.2^{\circ}$ and at this point could not progress any further. Work to find solutions inside the required range was often muddled, although some candidates were able to find one of the two solutions required for $x$. Only a minority of candidates were able to find both solutions correctly, but a number of these candidates were penalised 1 mark by offering at least one extra solution in the required range.

In part (b), many candidates were able to correctly substitute $1-\cos ^{2} x$ for $\sin ^{2} x$, and manipulate their resulting equation to find a correct quadratic equation in $\cos x$, with a few candidates either making sign or bracketing errors. It was disappointing, however, to see a fair number of candidates who thought that $\cos x$ could be replaced by $1-\sin x$ in the initial equation and then went on to attempt to solve a quadratic equation in $\sin x$. Although the majority were able to factorise $2 \cos ^{2} x+7 \cos x-4$ to give $(2 \cos x-1)(\cos x+4)$, a minority incorrectly factorised to give $(2 \cos x+1)(\cos x-4)$. Many candidates went on to solve $\cos x=\frac{1}{2}$ to give $x=\frac{\pi}{3}$, but the second solution sometimes ignored or incorrectly found. A minority of candidates worked in degrees, but most gave their answers in radians in terms of $\pi$.

## Question 9

Many candidates had difficulty in their attempts to establish the given result for the surface area in part (a) of this question. Solutions often consisted of a confused mass of formulae, lacking explanation of whether expressions represented length, area or volume. Formulae for arc length and sector area usually appeared at some stage, but it was often unclear how they were being used and at which point the substitution $\theta=1$ was being made. It was, however, encouraging to see well-explained, clearly structured solutions from good candidates.

Having struggled with part (a), some candidates disappointingly gave up. The methods required for the remainder of the question were, of course, more standard and should have been familiar to most candidates.

In part (b), most candidates successfully differentiated the given expression then formed an equation in $r$ using $\frac{\mathrm{d} S}{\mathrm{~d} r}=0$. While many solved $2 r-\frac{1800}{r^{2}}=0$ successfully, weaker candidates were sometimes let down by their algebraic skills and could not cope correctly with the negative power of $r$. A common slip was to proceed from $r^{3}=900$ to $r=30$.

In part (c), the majority of candidates correctly considered the sign of the second derivative to establish that the value of $S$ was a minimum, although occasionally the second derivative was equated to zero.

Those who proceeded as far as part (d) were usually able to score at least the method mark, except when the value of $r$ they substituted was completely inappropriate, such as the value of the second derivative.

## Statistics for C2 Practice Paper Gold Level G3

Mean score for students achieving grade:

| Qu | Max <br> score | Modal <br> score | Mean <br> \% | ALL | A* $^{*}$ | $\mathbf{A}$ | B | C | D | E | $\mathbf{U}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 6 |  | 78 | 4.68 |  | 5.70 | 5.43 | 5.10 | 4.60 | 3.77 | 1.98 |
| $\mathbf{2}$ | 7 | 7 | 72 | 5.05 | 6.73 | 6.41 | 5.86 | 5.36 | 4.77 | 4.12 | 2.22 |
| $\mathbf{3}$ | 7 |  | 66 | 4.62 |  | 6.35 | 4.91 | 4.39 | 3.51 | 2.90 | 1.66 |
| $\mathbf{4}$ | 7 |  | 57 | 3.99 |  | 5.90 | 4.33 | 3.32 | 2.20 | 1.61 | 0.73 |
| $\mathbf{5}$ | 8 |  | 57 | 4.59 |  | 6.73 | 4.83 | 3.73 | 2.47 | 2.10 | 1.24 |
| $\mathbf{6}$ | 8 |  | 51 | 4.09 |  | 6.53 | 4.31 | 3.21 | 1.97 | 1.26 | 0.54 |
| $\mathbf{7}$ | 9 |  | 61 | 5.50 |  | 7.47 | 6.37 | 5.43 | 4.56 | 3.87 | 2.31 |
| $\mathbf{8}$ | 10 |  | 56 | 5.61 | 9.79 | 9.00 | 7.40 | 5.72 | 3.89 | 2.26 | 0.64 |
| $\mathbf{9}$ | 13 |  | 45 | 5.91 |  | 11.21 | 7.78 | 5.05 | 2.82 | 1.37 | 0.31 |
|  | $\mathbf{7 5}$ |  | $\mathbf{5 8 . 7 2}$ | $\mathbf{4 4 . 0 4}$ | $\mathbf{1 6 . 5 2}$ | $\mathbf{6 5 . 3 0}$ | $\mathbf{5 1 . 2 2}$ | $\mathbf{4 1 . 3 1}$ | $\mathbf{3 0 . 7 9}$ | $\mathbf{2 3 . 2 6}$ | $\mathbf{1 1 . 6 3}$ |

